Modeling Blur in X-ray Radiography using a Systems Approach

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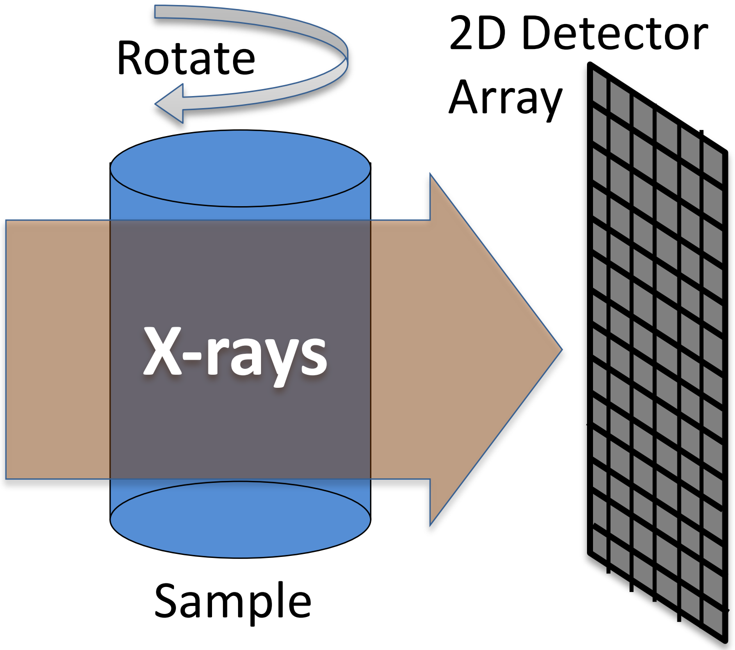
May 23, 2018

LLNL-PRES-751512
This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

Background: X-ray Radiography & Computed Tomography



- **Radiography:**
  - A 2D projected image of a 3D sample
  - Projected image is also called a “Radiograph”
  - Detector array measures the intensity of x-rays incident on it
- **Computed Tomography (CT)**
  - Radiographs at multiple rotation angles of the sample
  - Sample is rotated while the x-ray source and detector stay fixed
  - 3D sample is reconstructed from all the radiographs

Motivation, Objective, & Impact

- **Motivation**
  - Blur results in inaccurate localization of sample edges
  - Total blur is the combination of blur from multiple sources: x-ray source, detector array, system motion, and object scatter.
- **Objective**
  - Estimate the point spread functions (PSF) of each individual source of blur
  - Use a data driven approach
- **Impact**
  - Reduce blur by
    - Upgrading the imaging system components causing the blur
    - Use deblurring algorithms to remove blur

The Basic X-ray Transmission Model

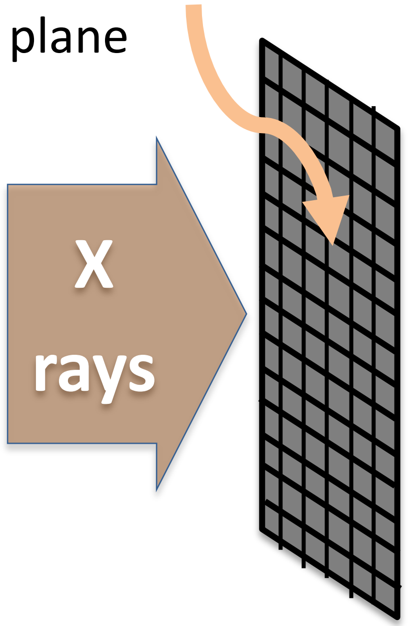
- **Beer's law:** Ratio of x-ray intensity with the sample, $I(r)$, and intensity without the sample, I_0, is equal to the negative exponential of the product of the total cross-section $\mu_{tot}(r)$, the sample density D, and the sample thickness L.

$$\frac{\boldsymbol{I(r)}}{\boldsymbol{I_0}} = \boldsymbol{I_N(r)} = \boldsymbol{e^{-\mu_{tot}(r)DL}}$$

- $\mu_{tot}(r)$ accounts for the loss of photons due to phenomena such as the photoelectric absorption, scatter, etc.

- **Drawback:** This model only accounts for the photons that emerge from the sample without any material interaction.

$r = (x, y)$ is position vector in 2D detector plane



2D Detector Array

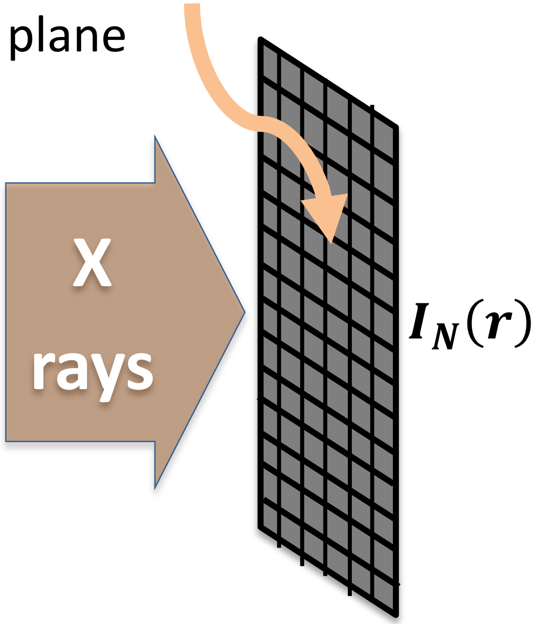
Quick Introduction to our X-ray Transmission Model

- Our Model: Beer’s law + First Order Coherent Scatter

- Transmission Model: Let $\mu_C(r)$ be the coherent scatter cross-section, then,

$$\boldsymbol{I_N(r) = T(r) = \underbrace{e^{-\mu_{tot}(r)DL}}_{\text{Photons that don't interact with the sample}} + \underbrace{C(\mu_{tot}, \mu_C, D, L)}_{\text{Single coherent scatter photons}} \circledast \underbrace{p_{cd}(r)}_{\text{Convolution with scatter PSF}}}$$

$r = (x, y)$ is position vector in 2D detector plane



2D Detector Array

- $p_{cd}(r)$ is the PSF of the blur due to coherent single scatter
  - A single parameter exponential density distribution
  - Models scatter as a function of x-ray energy and scatter angle

Blur Models

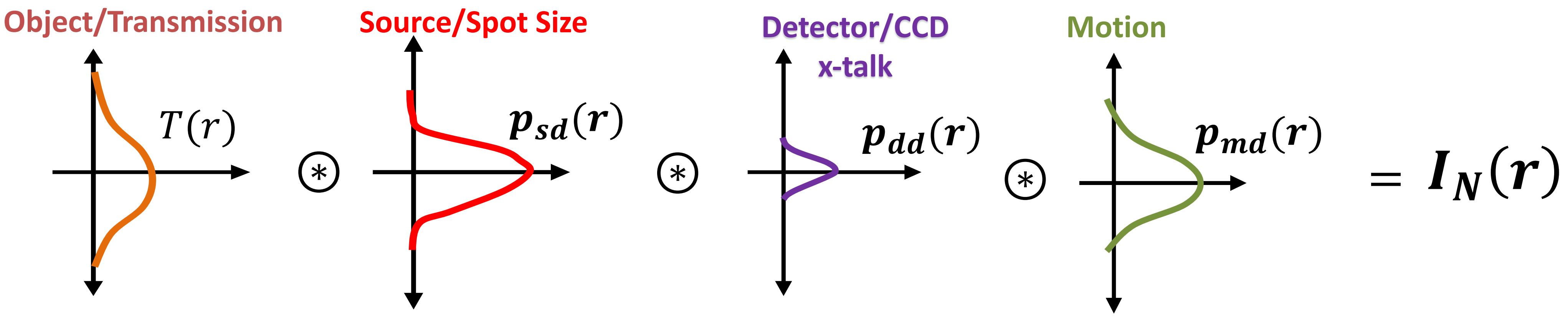
- Due to blur from the x-ray source, detector, and object motion, the normalized intensity at the detector plane is a convolution of multiple PSFs,

Blur PSF due to detector

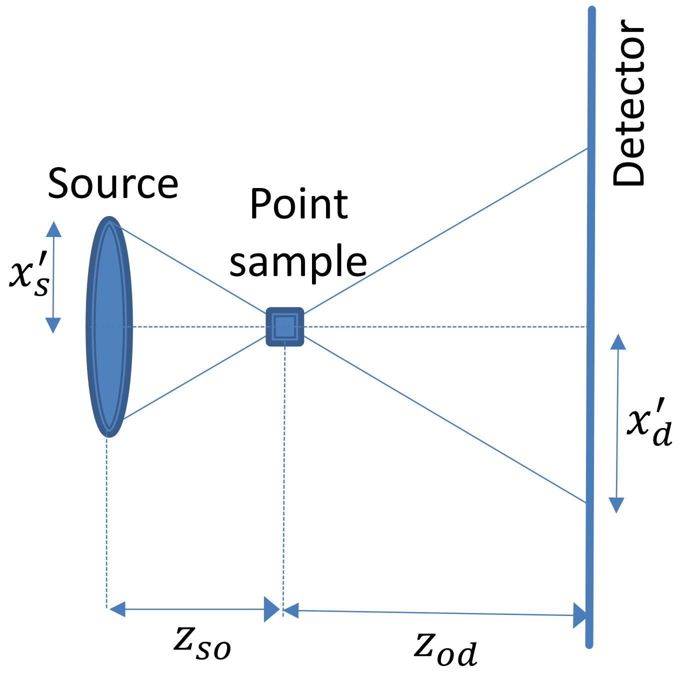
$$I_N(r) = T(r) \circledast \boldsymbol{p_{sd}(r)} \circledast \boldsymbol{p_{dd}(r)} \circledast \boldsymbol{p_{md}(r)}$$

Blur PSF due to source at detector plane

Blur PSF due to system motion at detector plane



Blur from the X-ray Source



By property of similar triangles, we have,

$$\frac{x'_d}{x'_s} = -\frac{z_{od}}{z_{so}}$$

- Source blur at the source plane –

$$p_{ss}(x'_s, y'_s) = \frac{1}{Z}\exp\left(-\frac{0.693}{W_s}\sqrt{x'^2_s + y'^2_s}\right)$$

- Source blur at the detector plane –

$$p_{sd}(x'_d, y'_d) = \frac{1}{Z}\exp\left(-\frac{0.693}{W_s}\frac{z_{so}}{z_{od}}\sqrt{x'^2_d + y'^2_d}\right)$$

where W_s is the full width half maximum (FWHM) of the source, z_{so} is the source to object distance, z_{od} is the object to detector distance, and Z is normalizing constant

Blur due to Detector and System Motion

- Blur due to detector and system motion do not vary with the source to object or object to detector distances.
- Hence, we combine the two effects and model the convolution of the PSFs due to detector and motion using a single exponential mixture density distribution.

$$\boldsymbol{p_{dd}}(\boldsymbol{r}) \circledast \boldsymbol{p_{md}}(\boldsymbol{r}) = p\frac{1}{Z_1}\exp\left(-\frac{0.693}{W_{d1}}\sqrt{x_d'^2 + y_d'^2}\right) + (1-p)\frac{1}{Z_2}\exp\left(-\frac{0.693}{W_{d2}}\sqrt{x_d'^2 + y_d'^2}\right)$$

Mixture parameter

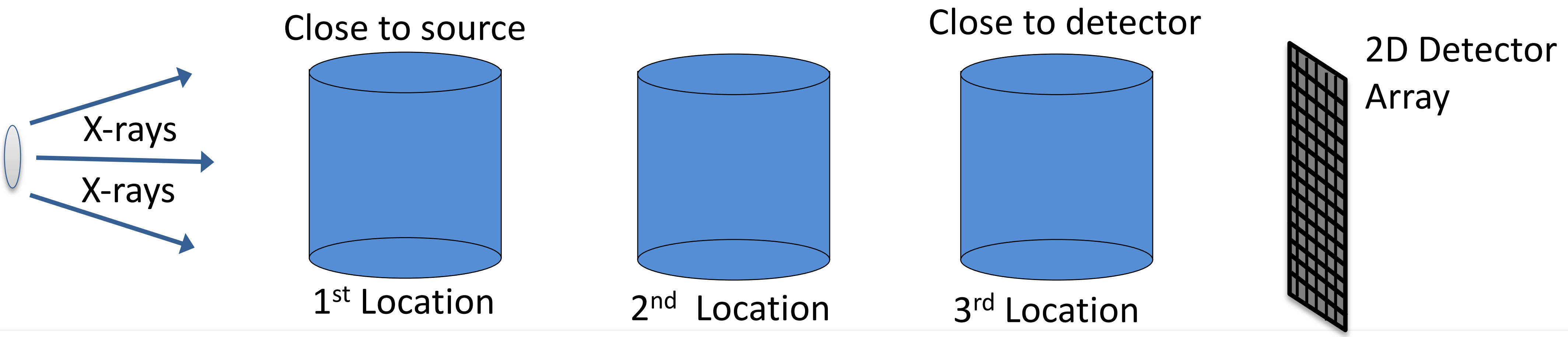
FWHM of first exponential density with short tail

FWHM of second exponential density with long tail

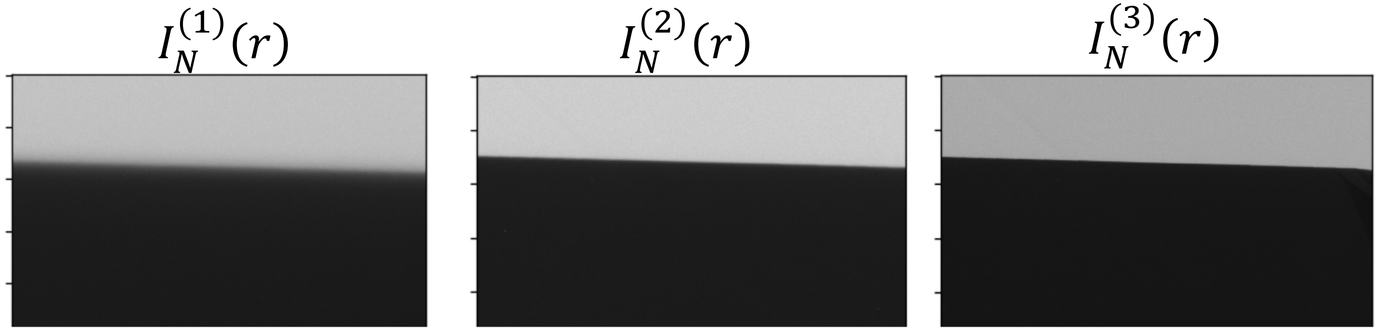
Z_1 and Z_2 are normalizing constants

Radiographs at Multiple Object Locations

- Changing the x-ray source, object, and detector positions will change the full width half maximums (FWHM) of each PSF by different amounts.
  - Source FWHM is proportional to the ratio of object to detector distance and source to object distance
- Acquire radiographs at different object to detector distances but fixed source to detector distance.
- Determine the sample width L, FWHMs W_s, W_{d1}, W_{d2}, and mixture probability p



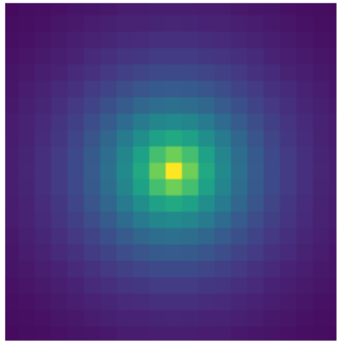
Data Driven Approach to PSF Estimation



Radiographs of an edge of a uniform width Tungsten plate at three different source to object distances.

Using numerical optimization

Image of a 2D PSF



Determine the width of the Tungsten sample L, FWHMs of the source PSF W_s and detector/motion PSFs W_{d1}, W_{d2}, and the mixture probability p of detector/motion PSF.

Optimization of Size of PSFs

- Find parameters that minimizes the following mean squared error –

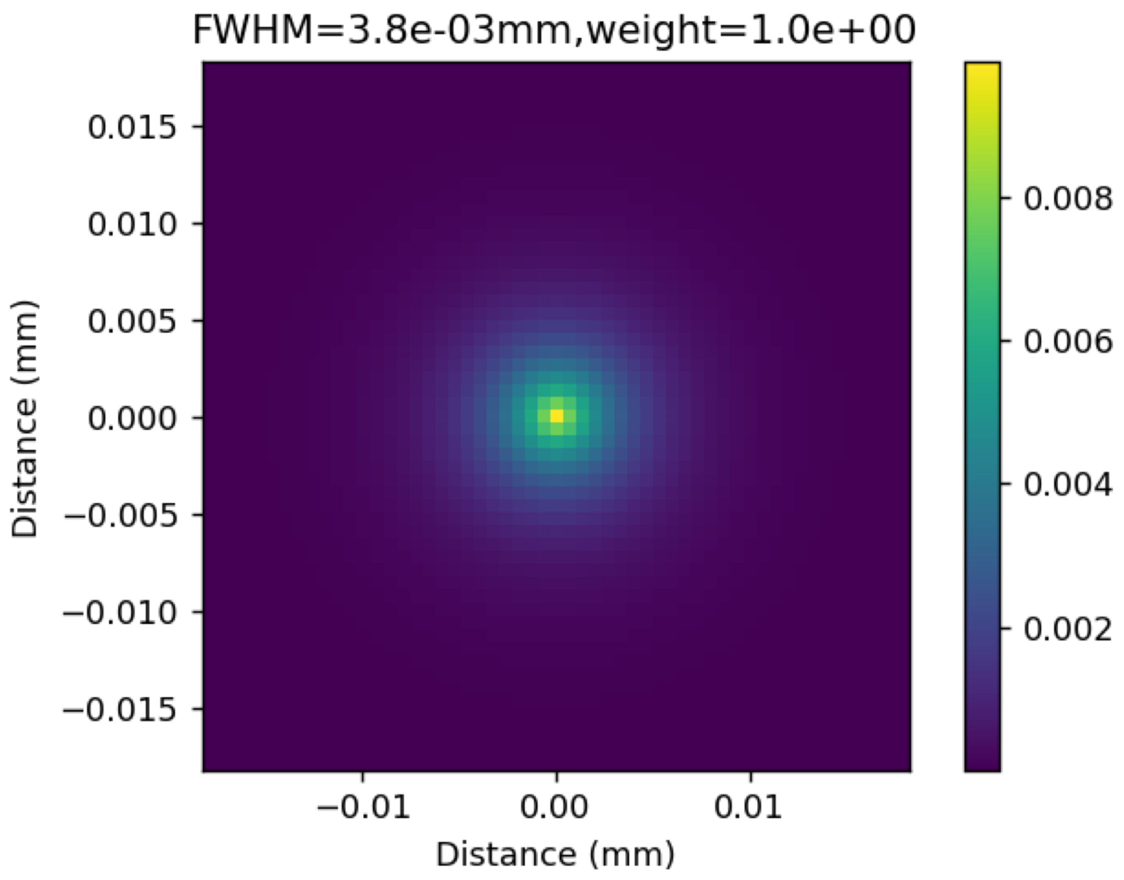
$$\left(\hat{L},\widehat{W}_s,p,\widehat{W}_{d1},\widehat{W}_{d2},\widehat{W}_c\right) = \underset{L,W_s,p,W_{d1},W_{d2},W_c}{\operatorname{argmin}}\left\{\sum_i \left\| I_N^{(i)}(r) - T^{(i)}(r)\circledast \boldsymbol{p}_{sd}^{(i)}(\boldsymbol{r})\circledast \boldsymbol{p}_{dd}^{(i)}(\boldsymbol{r})\circledast \boldsymbol{p}_{md}^{(i)}(\boldsymbol{r})\right\|^2\right\}$$

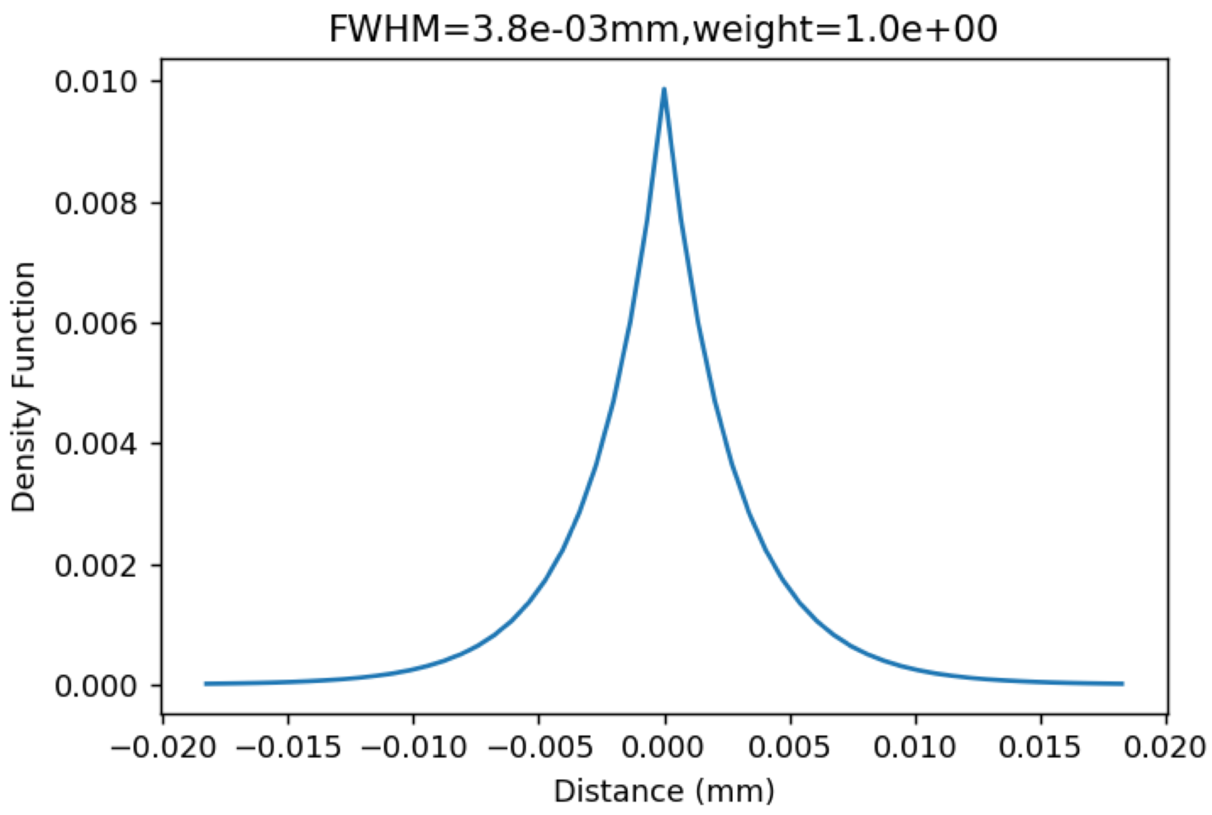
where $\boldsymbol{T}^{(i)}(\boldsymbol{r}) = \boldsymbol{e}^{-\boldsymbol{\mu}_{tot}(\boldsymbol{r})\boldsymbol{D}\boldsymbol{L}} + \boldsymbol{C}(\boldsymbol{\mu}_{tot},\boldsymbol{\mu}_{C},\boldsymbol{D},\boldsymbol{L})\circledast \boldsymbol{p}_{cd}^{(i)}(\boldsymbol{r})$

- $\widehat{W}_s$ gives the FWHM estimate for source PSF
- $\widehat{W}_{d1},\widehat{W}_{d2}$ gives the FWHM estimates for detector and motion PSF
- $\widehat{W}_c$ gives the FWHM estimate for Coherent scatter

Beyond the scope of this presentation

Source PSF estimated by the Optimizer

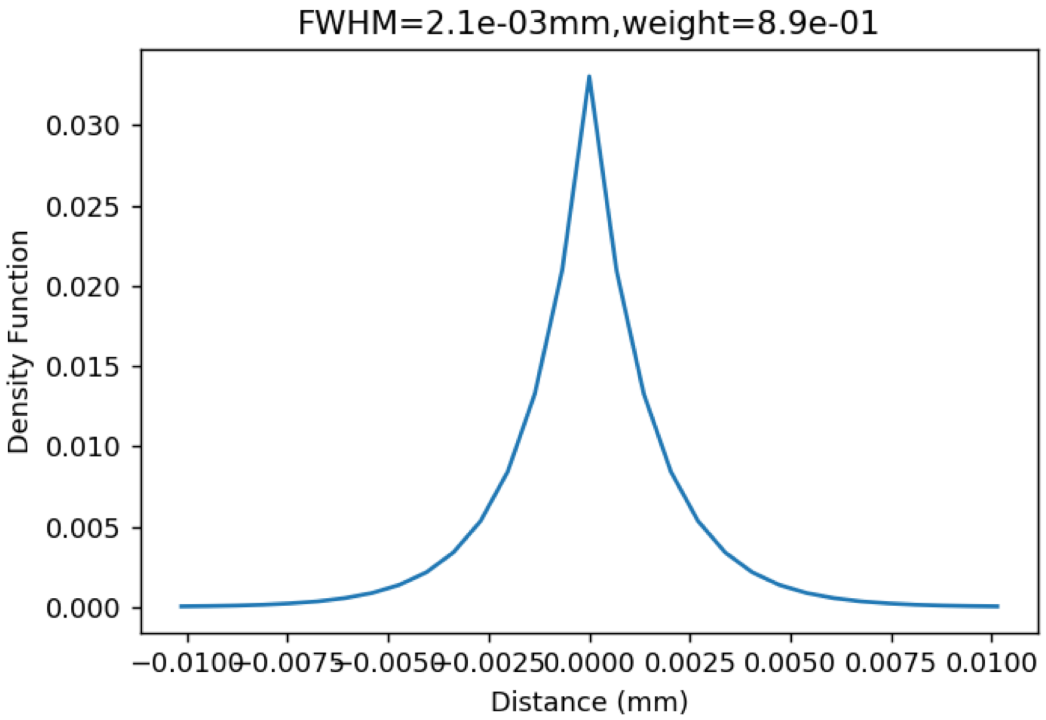




Agrees with manufacturer provided FWHM value of 4 micrometers.

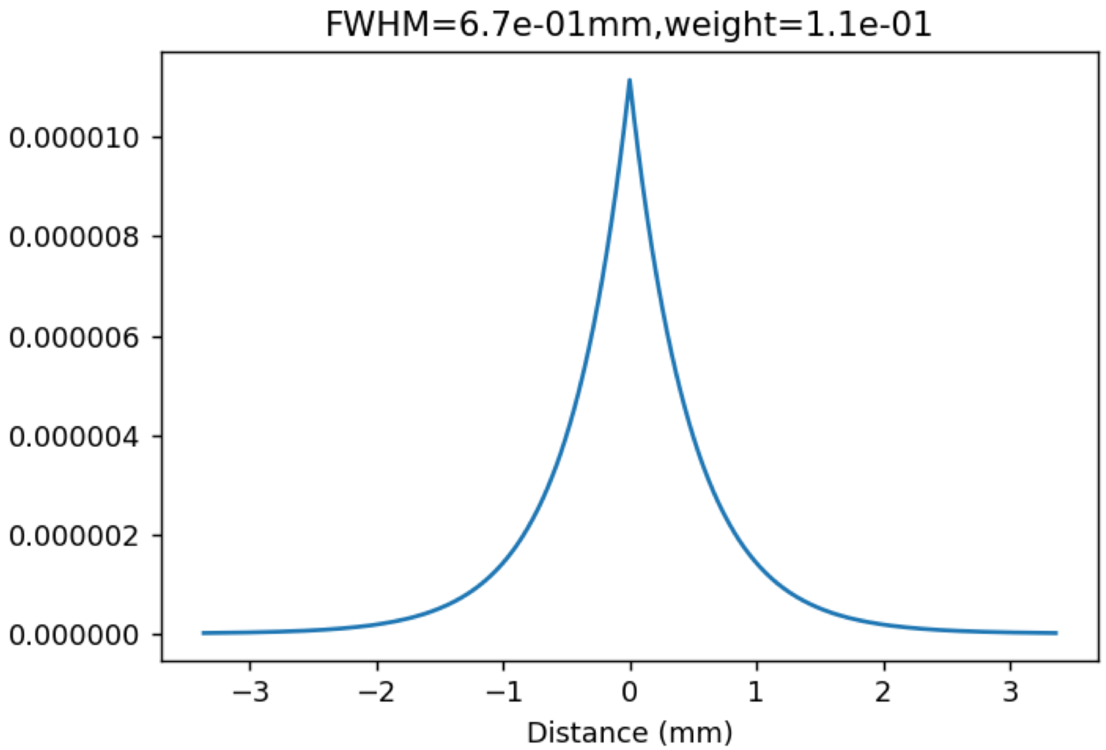
Combined Detector & Motion PSF
Only showing a 1D slice of PSF

Weight/probability $\mathbf{p = 0.89\times}$



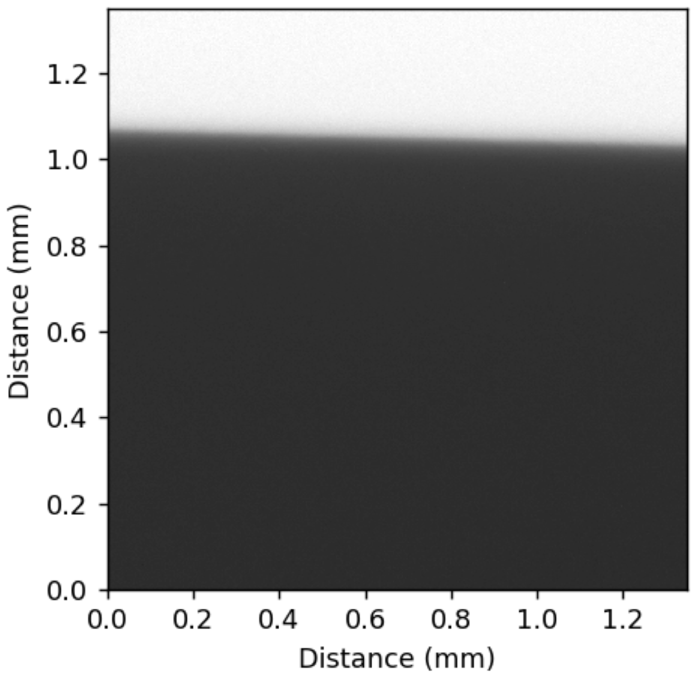
FWHM = $\mathbf{2.1\mu m}$

Weight/probability $\mathbf{(1-p) = 0.11\times}$

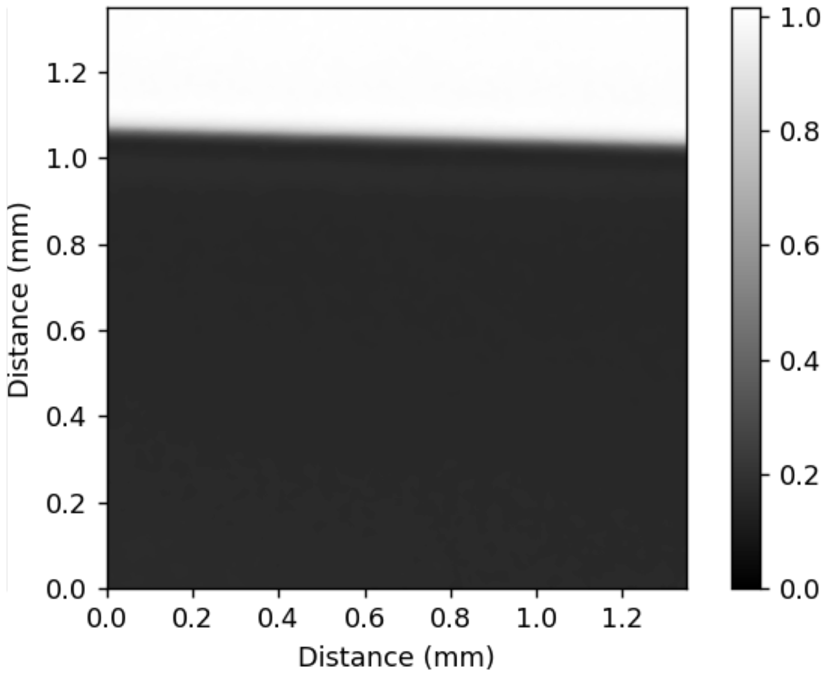


FWHM = $\mathbf{670\mu m}$

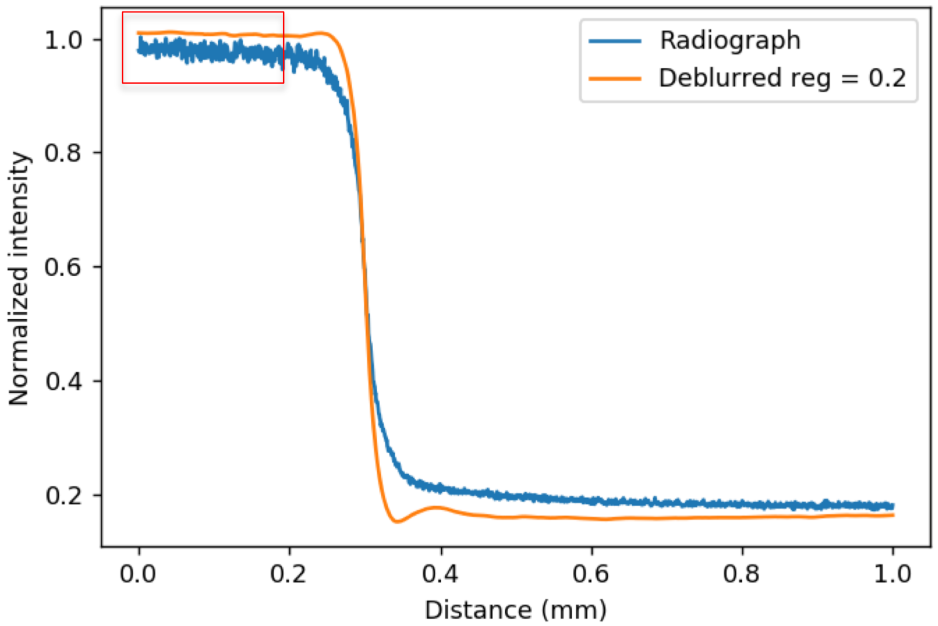
Deblur using a Regularized Iterative Least Squares Algorithm



Blurred Radiograph



Deblurred Radiograph



Line Profile comparing the deblurred result to the input radiograph

- Deblurring operation causes ringing artifacts
- Used regularization to reduce ringing
- Algorithm used is an iterative least squares technique with regularization

Noise Standard Deviation in red box

Input Radiograph: 0.0096

Deblurred Radiograph: 0.0017

Conclusions

- Data driven approach to model and estimate blur
- Useful to determine both spatially variant and invariant blur
- Use optimization to determine blur shape and size
- Reduce blur by
  - Upgrading the imaging system components causing the blur
  - Use deblurring algorithms to remove blur
- To use these techniques, contact me at **mohan3@llnl.gov**

Thank you!
Questions?

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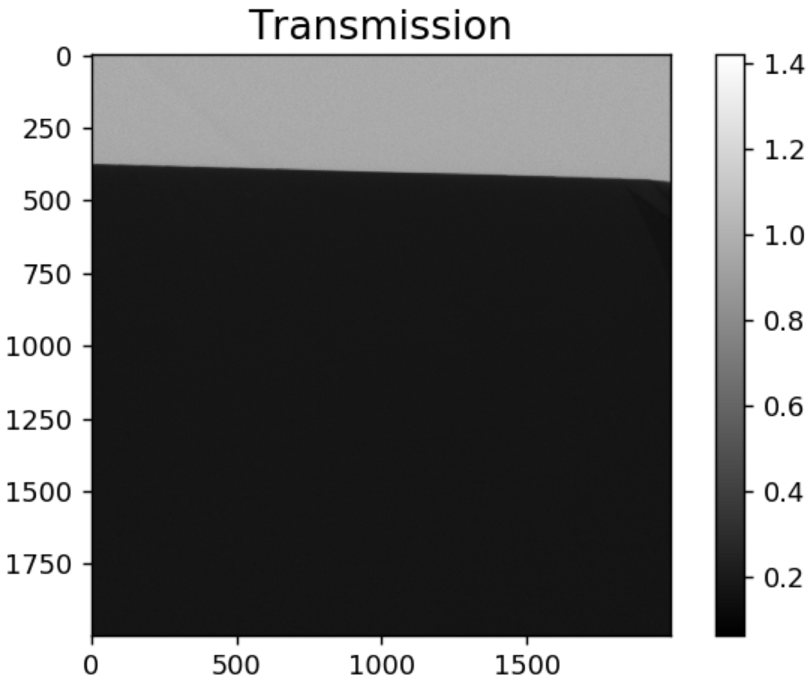
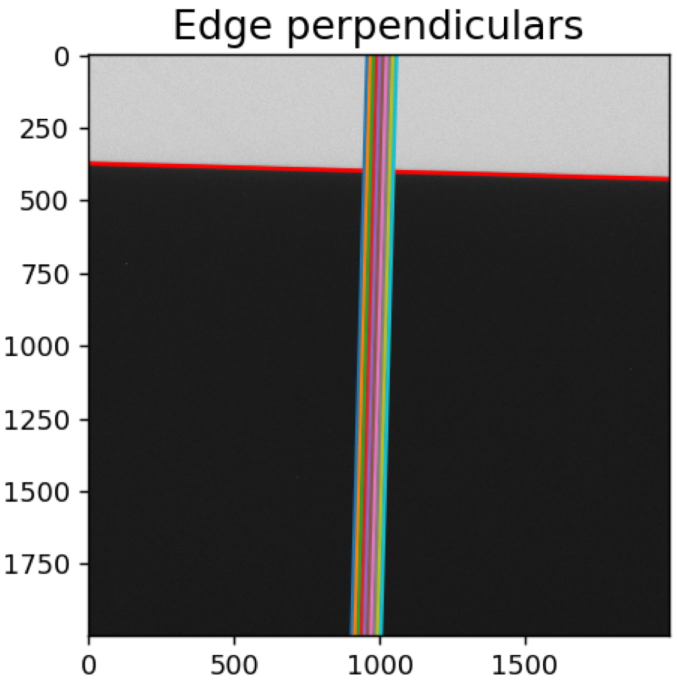
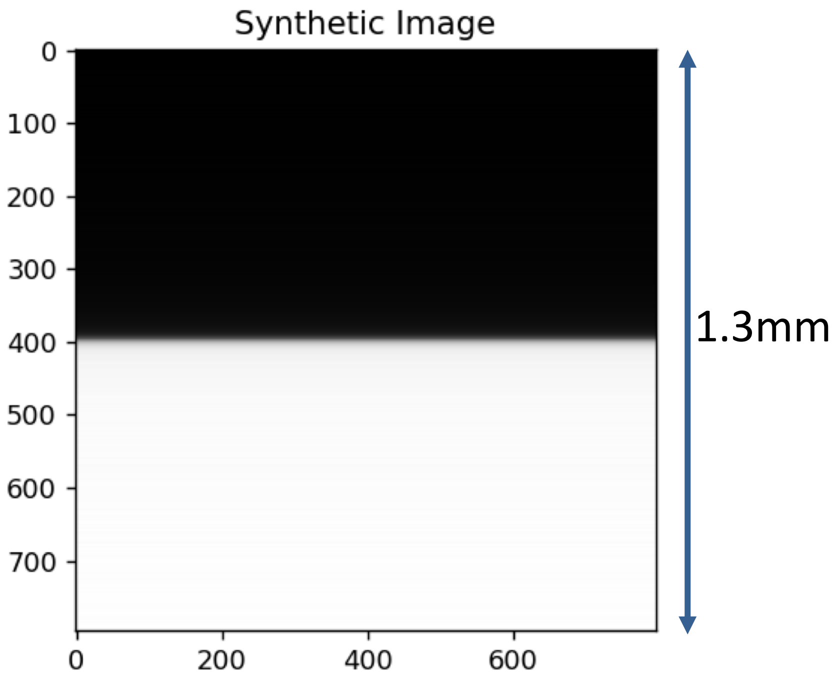
Extracting the Edge Line Profiles of a Tungsten Plate



Normalized intensity of a Tungsten edge radiograph



Detect Tungsten edge, draw perpendiculars to edge, and extract line profiles across the edge



Align and average the line profiles to reduce noise. Generate a lower noise image by repeating the line profile horizontally.

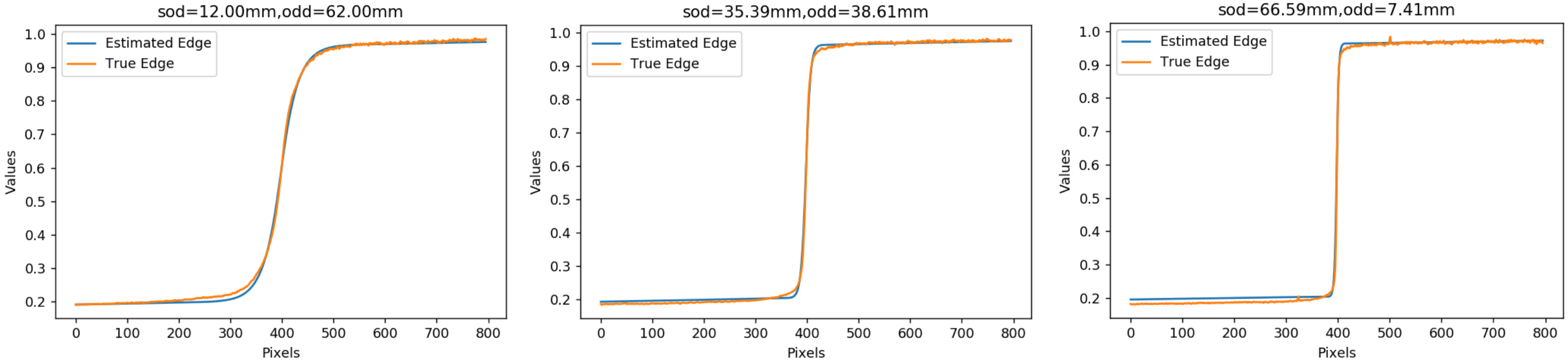
Our X-ray Transmission Model (Detailed)

- Our Model: Beer's law + First Order Coherent Scatter
- Transmission Model: Let $\mu_C(r)$ be the coherent scatter cross-section, then,

$$\boldsymbol{I_N(r) = T(r) = \underbrace{e^{-\mu_{tot}(r)DL}}_{\text{Photons that don't interact with the sample}} + \underbrace{\left(1 - e^{-\mu_{tot}(r)DL}\right)}_{\text{Photons that interact with the sample}} \underbrace{e^{-\mu_{tot}(r)DL/2}}_{\text{Reabsorption of photons after scatter}} \underbrace{\frac{\mu_c(r)}{\mu_{tot}(r)}}_{\text{Fraction of coherent scatter photons}} \underbrace{\circledast\, p_{cd}(r)}_{\text{Convolution with scatter PSF}}}$$

- $p_{cd}(r)$ is the PSF of the detector blur due to coherent single scatter
  - A single parameter exponential density distribution
  - Models scatter as a function of x-ray energy and scatter angle

Data Fit Quality after Optimization



True edge is a line profile of $I_N(r)$

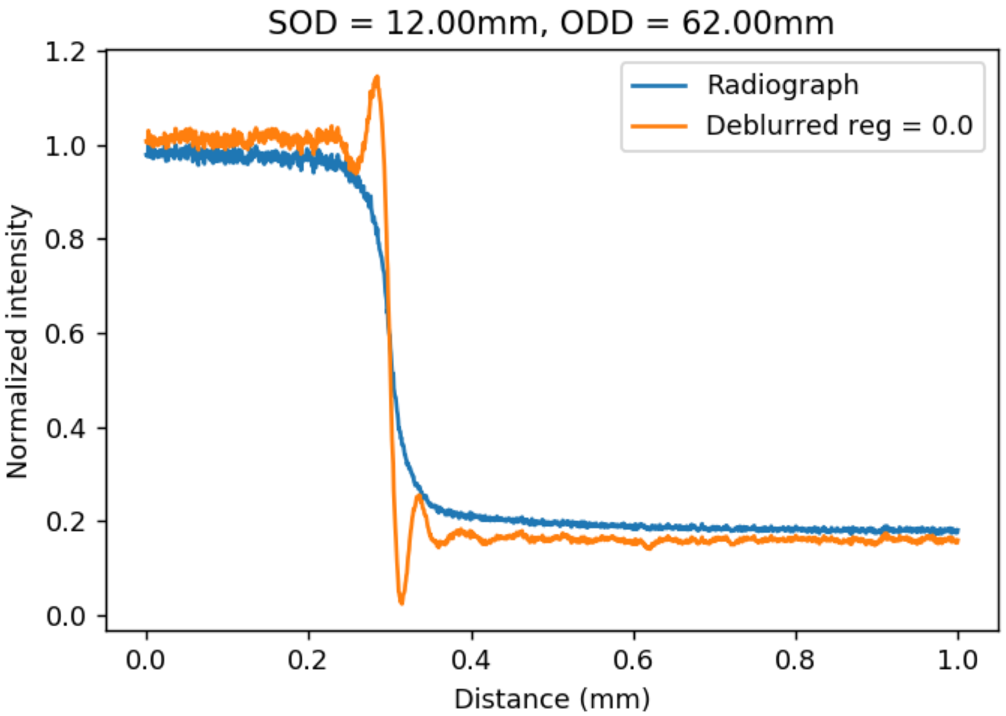
Estimated edge is a line profile of $T(r) \circledast \boldsymbol{p_{sd}(r)} \circledast \boldsymbol{p_{dd}(r)} \circledast \boldsymbol{p_{md}(r)}$

Regularized Least Squares Iterative Deblurring Algorithm (Hidden Slide)

- Deblurred reconstructing of sample is given by,
- $\hat{T}(r) = \mathrm{argmin}_{T(r)} \left\{ \left\| I_N(r) - T(r) \circledast \boldsymbol{p_{sd}}(\boldsymbol{r}) \circledast \boldsymbol{p_{dd}}(\boldsymbol{r}) \circledast \boldsymbol{p_{md}}(\boldsymbol{r}) \right\|_{\Lambda}^{2} + R(x) \right\}$

$R(x) \rightarrow L_{1.2}$ regularization

$\Lambda \rightarrow$ Noise matrix modeling Poisson noise in radiograph



Ringing when regularization parameter is 0

Noise Standard Deviation in red box
Input Radiograph: 0.0096
Deblurred Radiograph: 0.0104